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Drag over Contaminated Fluid Sphere with Slip Condition

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Abstract— In this manuscript stokes viscous flow past a partially contaminated fluid sphere with interfacial slip condition is considered. No mass transfer, zero tangential velocity on contaminated part, interfacial slip condition over clear part, shear stress continuity on clear part and regularity condition at far away from the body are considered for boundary conditions for evaluation of stream function. Drag on the contaminated fluid sphere is evaluated. As a special case (i) fluid sphere with no slip condition, (ii) sphere with no slip conditions are also deduced from the obtained results.

Index Terms— Slip condition, Gegenbauer polynomials, cap.

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1 Introduction

Study of flow in drops has wide applications in natural and engineering such as nuclear reactors, internal combustion engines, sediment and pollutant transport processes.

Stokes [1] solved the creeping flow problem past a sphere neglecting inertial terms in the Navier Stokes equations. Basset [2] calculated drag over a sphere in terms of slip parameter s (Trostel number), $s = \frac{\beta a}{\mu}$. Creeping flow over a fluid sphere was studied analytically independently by Rybczynski [3] and Hadamard [4]. Happel and Brenner [5] discussed creeping flow past a sphere with no slip boundary condition. Clift et.al., [6], Michaelides [7] in their monographs discussed about viscous flow past a fluid sphere with no slip boundary condition. Sadhal and Johnson [8] derived exact solution for drag force for a fluid sphere with stagnation cap over its boundary along the rear side of the fluid drop. Stagnation cap is the collection of surfactant at the rear side on the surface of the fluid sphere. Saboni [9] numerically discussed about the contamination effects on a fluid sphere for Reynolds numbers from 0.1 to 400 and viscosity ratio of external to internal fluids ranging from 0 to 10 at different stagnation cap angle.

Feng et al., [10] has discussed about the viscous flow past a viscous drop with interfacial condition at small but finite Reynolds numbers. He derived a formula for drag coefficient. As special cases he derived an expression for drag for solid sphere with (i) interfacial slip condition, (ii) no slip boundary condition.

In the present study the drag force for stokes flow for a viscous fluid over a contaminated fluid sphere is considered with the interfacial slip over the clear part. Stream function and drag over the contaminated fluid sphere are derived. The stream lines and vorticity lines drawn for different values of viscosity ratio μ, slip parameter value (s).

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2 Formulation of the Problem

A fluid sphere of radius a with contamination at rear end, which is held fixed in an uniform viscous flow, is considered. The fluid within the sphere and fluid flowing past the sphere are assumed to be immiscible. The flow is steady, incompressible, axi symmetric, with uniform velocity u_0 . A spherical polar coordinate system is considered with origin at the center of the sphere and Z-axis along the direction of uniform flow. $\mu_i, \mu_e, \rho_i, \rho_e$ are viscosity, density of interior and exterior fluids.

The viscosity ratio is taken as $\mu = \frac{\mu_i}{\mu_e}, \rho = \frac{\rho_i}{\rho_e}$.

The viscous fluid is assumed to flow from left to right. In the fluid sphere the clear part (no cap region) is considered for $-1 < x < x_0$ and contaminated part (cap region) is for $x_0 < x < 1$ where $x = \cos\theta$. x_0 is the position of cap or cosine angle of contamination.

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The velocity components in radial direction U and transverse direction V can be expressed in terms of stream function as

$$U(R,\theta) = \frac{1}{R^2 \sin\theta}\frac{\partial \Psi}{\partial \theta}, \quad V(R,\theta) = \frac{-1}{R\sin\theta}\frac{\partial \Psi}{\partial R} \quad (2.1)$$

Any physical (dimensional or non dimensional) quantity for internal flow is represented by fi and external flow by fe.

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The following

$$R = ar, \quad \bar{Q} = U_0 \bar{q}, \quad U = U_0 u, \quad V = V_0 v, \quad \Psi = a^2 U_0 \psi$$

Reynolds number for external flow is $Re = \frac{2a\rho_e U_0}{\mu_e}$

and for internal flow $Rei = \frac{Re\rho}{\mu}$

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is used for casting equations into non dimensional form.

The equation of motion in terms of stream function for the internal flow is

$$E^4 \psi i = 0 \qquad (2.2)$$

The equation of motion in terms of stream function for the external flow is

$$E^4 \psi e = 0 \qquad (2.3)$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{(1-x^2)}{r^2}\frac{\partial^2}{\partial x^2} = Stokes\ stream\ function\ operator$$

Equations (2.2), (2.3) are solved for $\psi e, \psi i$ using the following boundary conditions (2.4)- (2.9):

$$At\ \theta = 0,\ \pi (or\ x = 1,\ x = -1)\ on\ r = 1,\ \psi e = \psi i = 0 \qquad (2.4)$$

Tangential velocity is zero along the contaminated part

$$\frac{\partial \psi e(1)}{\partial r} = \frac{\partial \psi i(1)}{\partial r} = 0 \quad for\ x_0 < x < 1 \qquad (2.5)$$

At the interface, tangential velocity is proportional to the tangential shear along the clear surface which implies

$$\frac{\partial^2 \psi e(1)}{\partial r^2} = (s+2)\frac{\partial \psi e(1)}{\partial r} - s\frac{\partial \psi i(1)}{\partial r} \quad for\ -1 < x < x_0 \qquad (2.6)$$

where s is the slip parameter given by $s = \frac{\beta a}{\mu}$
(β is coefficient of sliding friction)

The shear stress is continuous along the clear surface at the interface which implies

$$\frac{\partial}{\partial r}\Big(\frac{1}{r^2}\frac{\partial \psi e}{\partial r}\Big) = \mu \frac{\partial}{\partial r}\Big(\frac{1}{r^2}\frac{\partial \psi i}{\partial r}\Big) \quad for\ -1 < x < x_0 \qquad (2.7)$$

Far away from the sphere, regularity condition is taken

$$\lim_{r\to\infty} \psi e = \frac{1}{2} r^2 (1-x^2) \qquad (2.8)$$

The physical condition that is zero velocity at the origin is finite

$$\lim_{r\to\infty} \psi i < \infty (is\, taken\, as\, zero) \qquad (2.9)$$

3 Solution of the Problem

The velocity components u, v and stream function ψ for internal and external flows are given by

$$u = \begin{cases} ui & for\ 0 < R \le a \\ ue & for\ a \le R \le \infty \end{cases},$$

$$v = \begin{cases} vi & for\ 0 < R \le a \\ ve & for\ a \le R \le \infty \end{cases},$$

$$\psi = \begin{cases} \psi i & for\ 0 < R \le a \\ \psi e & for\ a \le R \le \infty \end{cases}$$

To match with uniform velocity at infinity, the solution for ψ can be assumed in the form:

$$\psi e(r,x) = \begin{cases} fen(r) G_2(x) & for\ -1 < x < x_0\ (no\ cap\ region) \\ fec(r) G_2(x) & for\ \ x_0 < x < 1\ \ (cap\, region) \end{cases}$$

$$\psi i(r,x) = \begin{cases} fin(r) G_2(x) & for\ \ -1 < x < x_0\ \ (no\, cap\, region) \\ fic(r) G_2(x) & for\ \ \ x_0 < x < 1\ \ (cap\, region) \end{cases}$$

where $G_2(x)$ is Gegenbauer polynomial of order 2.

The functions for external flow with no cap and cap regions are fen(r), fec(r) and for internal flow with no cap and cap regions are fin(r), fic(r) respectively which are given as

$$fen(0,2) = (A_1 r + \frac{B_1}{r} + C_1 r^2 + D_1 r^4),$$

$$fec(0,2) = (A_2 r + \frac{B_2}{r} + C_2 r^2 + D_2 r^4) \quad (3.1)$$

$$fin(0,2) = (A_3 r + \frac{B_3}{r} + C_3 r^2 + D_1 r^4),$$

$$fic(0,2) = (A_4 r + \frac{B_4}{r} + C_4 r^2 + D_4 r^4) \quad (3.2)$$

The arbitrary parameters values are evaluated substituting the conditions (2.4) to (2.9)

$$\psi e(r,x) = \begin{cases} fen(r)G_2(x) = (r^2 - (\frac{2s+3s\mu+6\mu}{2s+2s\mu+6\mu})r + \frac{s\mu}{2s+2s\mu+6\mu}\frac{1}{r})G_2(x) & for\ -1 < x < x_0 \\ fec(r)G_2(x) = (r^2 - (\frac{3s+6}{2s+6})r + \frac{s}{2s+6}\frac{1}{r})G_2(x) & for\ \ x_0 < x < 1 \end{cases} \qquad (3.3)$$

$$\psi i(r,x) = \begin{cases} fin(r)G_2(x) = (\frac{s}{2s+2s\mu+6\mu})(r^4 - r^2)G_2(x) & for\ -1 < x < x_0 \\ fic(r)G_2(x) = 0 & for\ \ x_0 < x < 1 \end{cases} \qquad (3.4)$$

The vorticity lines can be obtained from the equations

$$\zeta = -\frac{E^2 \psi}{r \sin\theta} \qquad (3.5)$$

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Taking swirl of vorticity as $\Omega = -E^2\Psi = \zeta r \sin\theta$,
we have

$$\Omega e(r,x) = \begin{cases} gen(r)G_2(x) & for \ -1 < x < x_0 \ (no\,cap\,region) \\ gec(r)G_2(x) & for \ \ x_0 < x < 1 \ (cap\,region) \end{cases} \tag{3.6}$$

$$\Omega i(r,x) = \begin{cases} gin(r)G_2(x) & for \ -1 < x < x_0 \ (no\ cap\ region) \\ gic(r)G_2(x) & for \ \ x_0 < x < 1 \ (cap\ \ region) \end{cases} \tag{3.7}$$

gen(r), gec(r), gin(r), gic(r) using (3.3) and (3.4)

$$\Omega e(r,x) = \begin{cases} gen(r)G_2(x) = (\dfrac{2s+3s\mu+6\mu}{s+s\mu+3\mu})\dfrac{1}{r}G_2(x) & for \ -1 < x < x_0 \\ gec(r)G_2(x) = (\dfrac{3s+6}{s+3})\dfrac{1}{r}G_2(x) & for \ \ x_0 < x < 1 \end{cases} \tag{3.8}$$

$$\Omega i(r,x) = \begin{cases} gin(r)G_2(x) = (\dfrac{5sr^2}{s+s\mu+3\mu})G_2(x) & for \ -1 < x < x_0 \\ gic(r)G_2(x) = 0 & for \ \ x_0 < x < 1 \end{cases} \tag{3.9}$$

4 DRAG

The drag force on the contaminated fluid sphere is evaluated using the following formula

$$Drag = 2\pi a^2 \int_0^{\pi} (T_{11}\cos\theta - T_{21}\sin\theta)\Big|_{r=a} \sin\theta d\theta \tag{4.1}$$

$$Drag = 2\pi a \mu u_0 \int_{-1}^{1} (T_{11}x - T_{21}\sqrt{1-x^2})\Big|_{r=a} \sqrt{1-x^2}\,dx \tag{4.2}$$

Stress tensor components are obtained as

$$T_{11} = -P + 2\mu\frac{\partial u}{\partial x} \quad and \quad T_{21} = \mu r\frac{\partial}{\partial r}(\frac{v}{r}) + \frac{\mu}{r}\frac{\partial u}{\partial\theta} \qquad x = \cos\theta \tag{4.3}$$

Substituting (4.3) in (4.2) and simplifying, gives drag value as

$$Drag = 2\pi a\mu u_0 [\frac{Re}{64}[-\frac{(s+3\mu)^2(1-x_0^2)^2}{(s+s\mu+3\mu)^2} + \frac{9(1-x_0^2)^2}{(s+3)^2}] + \frac{(2s+6\mu+s\mu)(1+x_0^3)}{\ldots} + \frac{(s+6)(1-x_0^3)}{\ldots}$$

(4.4)

The Drag Coefficient $C_D = \dfrac{Drag}{(\frac{1}{2}\rho U_0^2 \pi a^2)}$

$$\therefore C_D = \frac{8}{Re}[\frac{Re}{64}[-\frac{(s+3\mu)^2(1-x_0^2)^2}{(s+s\mu+3\mu)^2} + \frac{9(1-x_0^2)^2}{(s+3)^2}] + \frac{(2s+6\mu+s\mu)(1+x_0^3)}{(2s+2s\mu+6\mu)} + \frac{(s+6)(1-x_0^3)}{2s+6} + \frac{s\mu(3x_0 - x_0^3+2)}{2s+2s\mu+6\mu} + \frac{s(2-3x_0+x_0^3)}{2s+6}]$$

Special cases

(i) If $x_0 = -1$ and as $s \to \infty$ (cap for contamination covers the whole sphere and we get the case of a solid sphere)
The drag coefficient in this case for solid sphere i.e.,

$$C_D = \frac{24}{Re}$$

(ii) If $x_0 = 1$, there is no contamination i.e., the boundary entire reduces to the sphere has no cap i.e.,
we get the case of a fluid sphere with slip condition.
In this case the drag coefficient is given by

$$C_D = \frac{8}{Re}[\frac{2s+6\mu+3s\mu}{s+s\mu+3\mu}]$$

The expression matches with the drag coefficient value of a fluid sphere with interfacial slip condition given by Feng et al.,[10],

(iii) As the slip parameter, $s \to \infty$ we get the case of a fluid sphere with no slip case.
In this case the drag coefficient is

$$C_D = \frac{8}{Re}[\frac{2+3\mu}{1+\mu}]$$

The expression matches with the drag coefficient of a fluid sphere with no slip condition mentioned by Rybczynski [3] and Hadamard [4].

(iv) Again as $\mu \to \infty$, we get the case of a solid sphere

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case with no slip condition.

In this case the drag coefficient is given by

$$C_D = \frac{24}{Re}$$

this matches with the solid sphere case of Stokes [1].

5 RESULTS AND DISCUSSIONS

The results for drag coefficient are presented in the form of graphs and the results for stream line and vorticity lines are presented in the form of contour graphs.

Figure (1) shows the variation of drag coefficient C_D with slip parameter(s) values for different x_0 values at fixed Reynolds number (Re)=1. In Figure 1, the value of C_D increases with increment of slip parameter value for any length of the cap. However, the value of C_D asymptotically reaches the standard value $\frac{24}{Re}$ as slip parameter $s \to \infty$ (matches with no slip condition). From 1(a), 1(b), we can observe that as $\mu \to \infty$, for the drag coefficient difference between fluid sphere with cap and solid sphere disappears i.e., when the cap is like a solid surface the effect of clear fluid part on the drag will be negligible.

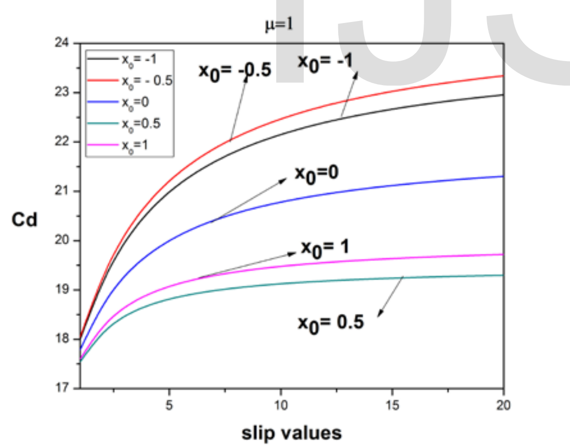


Fig. 1 (a). C_D graphs at different slip values and x_0 values with fixed Reynolds number at (Re) =1, μ = 1.

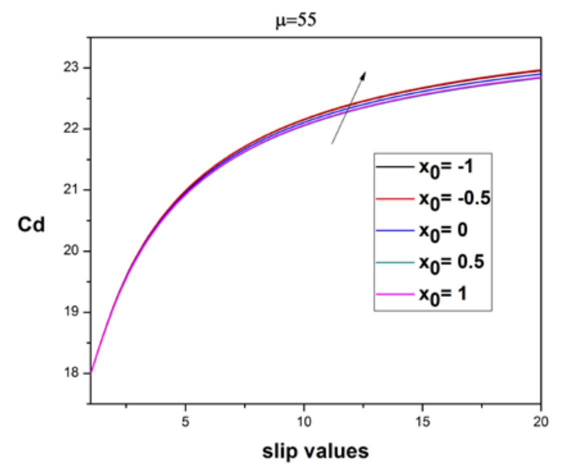


Fig. 1(b). C_D graphs at different slip values and x_0 values with fixed Reynolds number at (Re) =1 μ = 55.

Figure (2) shows the variation of C_D with x_0 for different values of slip parameter(s) at Re=1.

In Figure 2(a) for small values of μ, the value of C_D first increases and attains the maximum at x_0= -0.7, beyond that it decreases and attains minimum at x_0= 0.7. With further increment of x_0 the value of C_D increases x_0 =1.

From 2(b) we observe that for large values of μ, the range of C_D lies between 22 to 24 only for $s > 10$ and the curve is almost a straight line, indicating the negligible effect of contamination cap on drag.

Increasing and decreasing nature of C_D for small values of μ, may be due to sudden change of flow pattern due to contamination cap. The flow pattern can be seen in figures 4, 5 and 6. The jump in the stream lines is due to negligence of non linear terms. If the non linear convective terms are considered, the problem becomes difficult to solve and much attention is required to solve it.

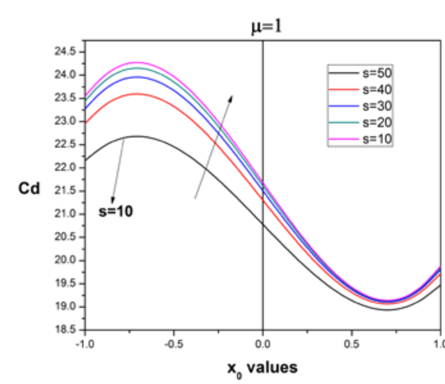


Fig. 2 (a) C_D graphs at different x_0 values, different slip parameters with fixed Reynolds number at (Re) =1, μ = 1.

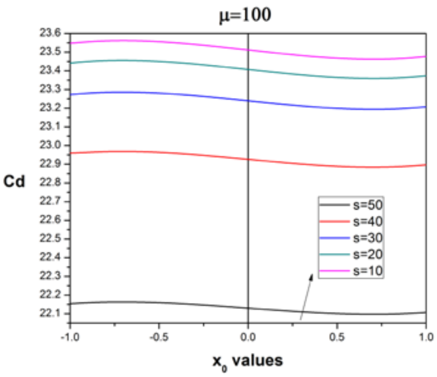


Fig. 2 (b) C_D graphs at different x_0 values,

Figure (3) shows the variation of C_D with slip parameter (s) value for different values of μ. The value of C_D increases rapidly with increment of slip for lower values of slip and asymptotically approaches the constant value as slip parameter $s \to \infty$. For higher value of μ the trend remains the same but magnitude of C_D increases for any given value of slip parmeter (s). For higher values of slip and μ the value of C_D matches with that of a solid sphere with no slip condition $\frac{24}{Re}$.

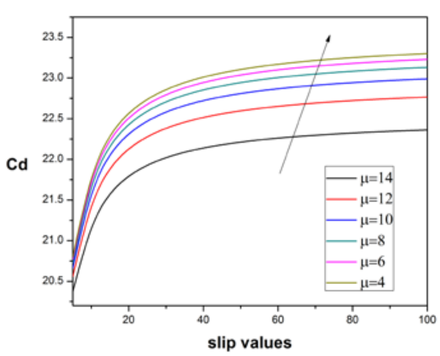
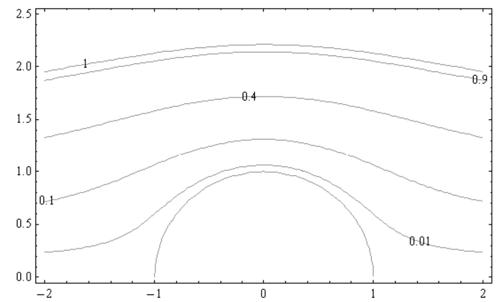


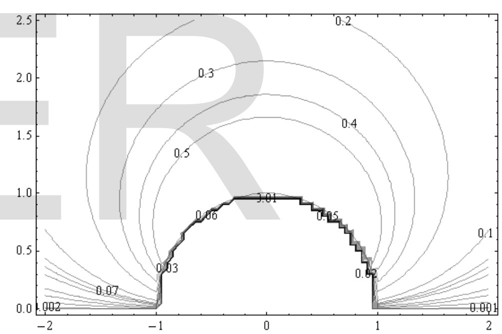
Fig. 3 C_D graph for different slip parameter(sp) values at x_0=0.3 with fixed Reynolds number (Re)=1.

Figure (4) represents the flow pattern at viscosity ratio (μ) =5, slip parameter value(s) =10, with varying of size of cap(x_0) from -1 to 1. At x_0 =1, where the fluid surface is covered entirely with cap, it behaves like a solid sphere with no internal circulations. The external stream line pattern is symmetric. At x_0= -0.7, internal circulations are observed for -1 < x < -0.7, the rest of the region (-0.7 < x < 1) is covered with cap, hence no internal circulation. With increase in $x_0$values,s the cap region is reduced and finally when x_0=1, there is no cap and it behaves like a full fluid sphere with internal circulation lines.



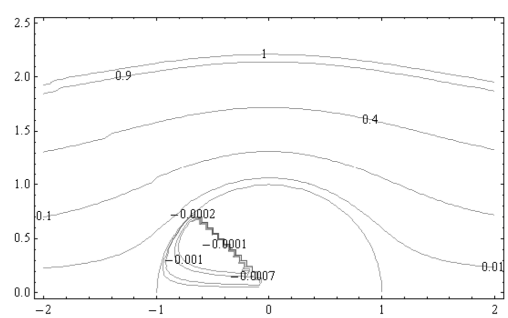
Stream lines at x_0 = -1

Fig. 4 (a). Stream lines for μ = 5, Slip (sp) =10, at X0 = -1 .



Vorticity lines at x_0= -1

Fig. 4 (b). Vorticity lines for μ = 5, Slip(sp)=10, at X0 = -1 .



Stream lines at x_0= -0.7

Fig. 4 (c). Stream lines for μ = 5, Slip (sp) =10, at X0 = -0.7 .

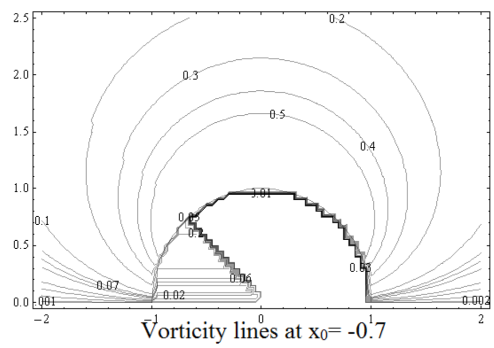


Fig. 4 (d). vorticity lines for μ = 5, Slip(sp)=10, at X0 = - 0.7 .

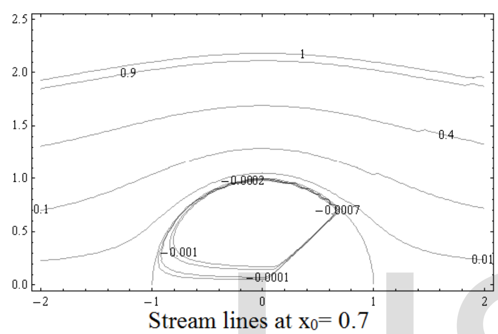


Fig. 4 (e). Stream lines for μ = 5, Slip (sp) =10, at X0 = 0.7 .

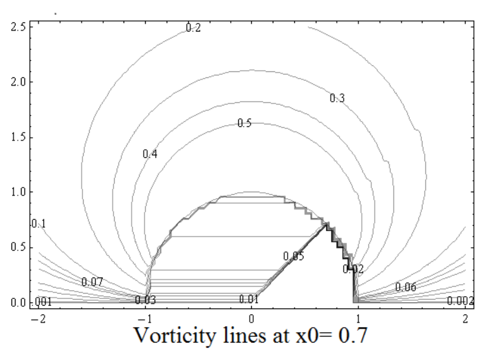


Fig. 4 (f). Vorticity lines for μ = 5, Slip(sp)=10, at X0 = 0.7 .

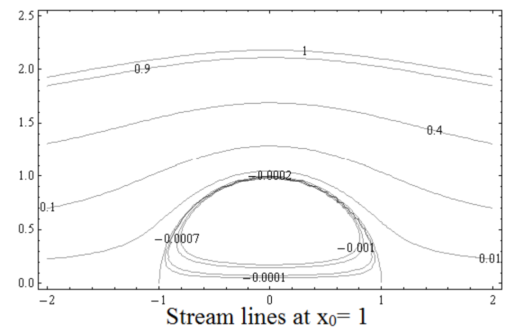


Fig. 4 (g). Stream lines for μ = 5, Slip (sp) =10, at X0 = 1 .

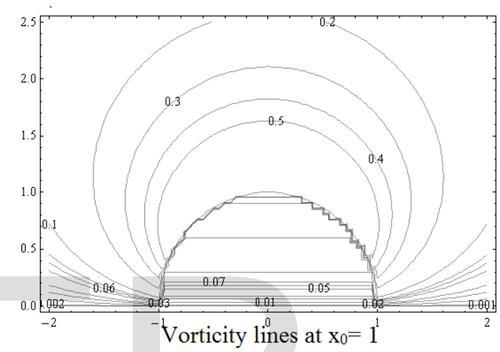


Fig. 4 (h). Vorticity lines for μ = 5, Slip(sp)=10, with X0 = 1.

Figure. 5 represents the flow pattern for different slip parameter(s) value with μ fixed at 30 and x_0=0.6. At slip=0 (perfect slip) the stream lines are in contact with the surface with no internal circulations. As the slip parameter values are increasing and tends to no slip case ($s \to \infty$) the stream lines are detaching from the surface.

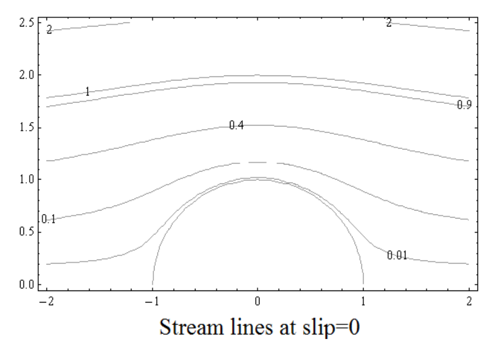


Fig. 5 (a).Stream lines for μ =30, x_0= 0. 6, Re=1 at slip s=0

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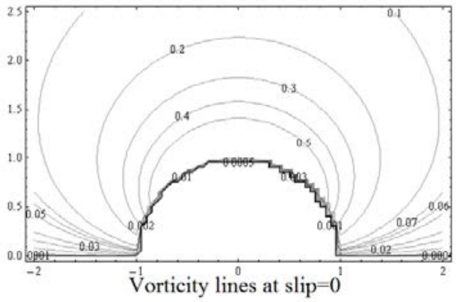

Fig. 5 (b).Vorticity lines for μ =30, x_0= 0.6, Re=1 at slip s=0

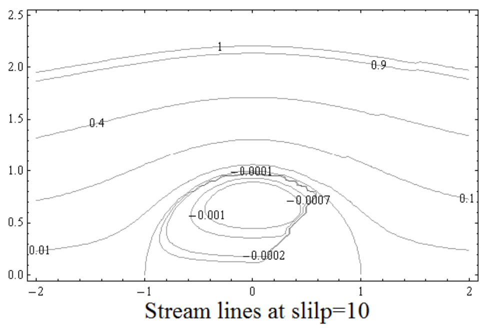

Fig. 5 (c). Stream lines for μ =30, x_0= 0.6, Re=1 at slip s=10

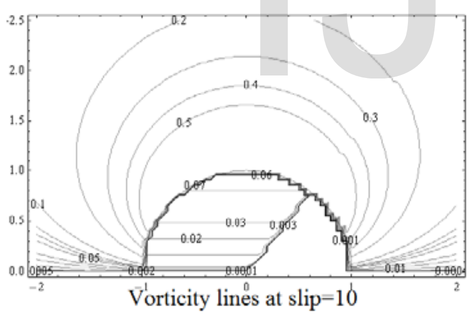

Fig. 5 (d).Vorticity lines for μ =30, x_0= 0.6, Re=1 at slip s=10

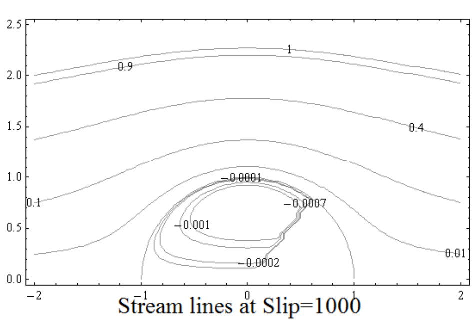

Fig. 5 (e). Stream lines for μ =30, x_0= 0.6, Re=1 at slip s=1000

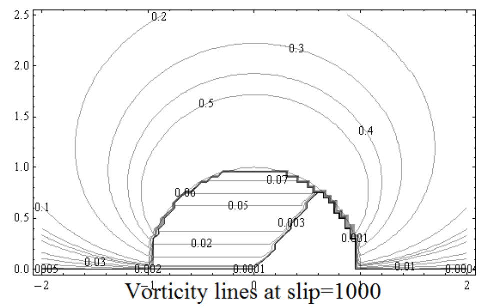

Fig. 5 (f).Vorticity lines for μ =30, x_0= 0.6, Re=1 at slip s=1000

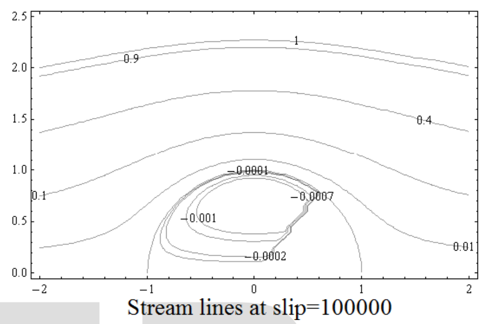

Fig. 5 (g). Stream lines for μ =30, x_0= 0.6, Re=1 at slip s=100000

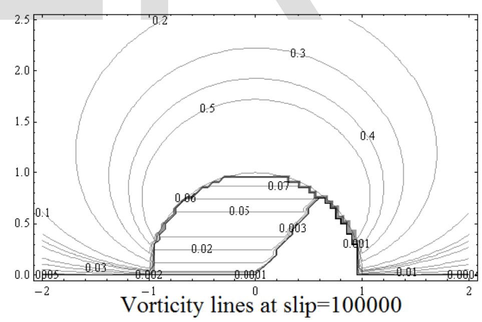

Fig. 5 (h).Vorticity lines for μ =30, x_0= 0.6, Re=1 at slip s=100000

Figure (6) represents the flow pattern for different values viscosity ratio (μ). It was observed that at low value of viscosity ratio μ there is a jump in the stream line pattern at the fluid sphere and solid sphere interface. This is reduced with increase in the μ value. With the increase in μ value,s the internal flow lines are reduced and at high value of μ the fluid sphere behaves like a solid one and hence there are no internal circulation lines.

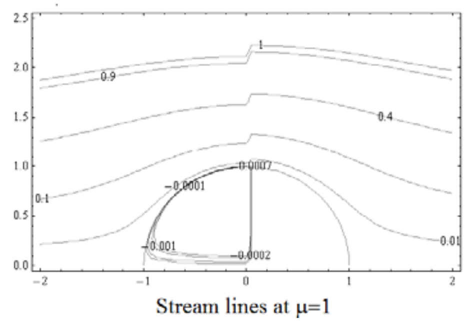

Fig. 6 (a). Stream lines for slip=10
$x_0 = 0$, Re=1 at μ =1

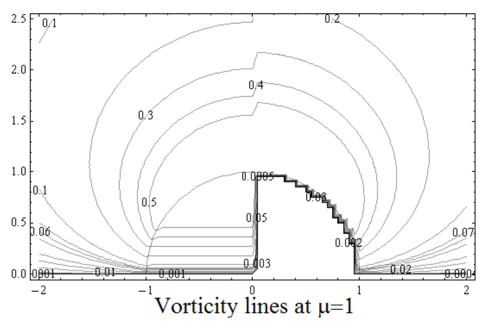

Fig. 6 (b). Vorticity lines for slip=10
$x_0 = 0$, Re=1 at μ =1

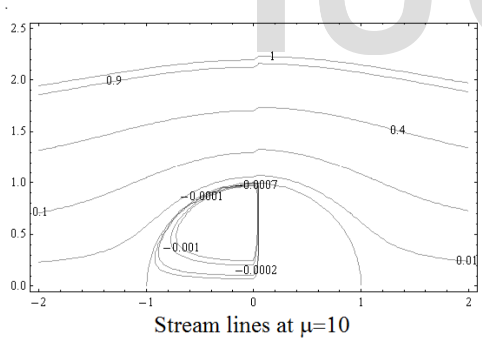

Fig. 6 (c).Stream lines for slip=10
$x_0 = 0$, Re=1 at μ =10

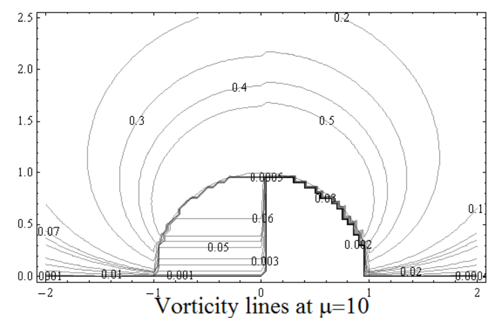

Fig. 6 (d). Vorticity lines for slip=10
$x_0 = 0$, Re=1 at μ =10

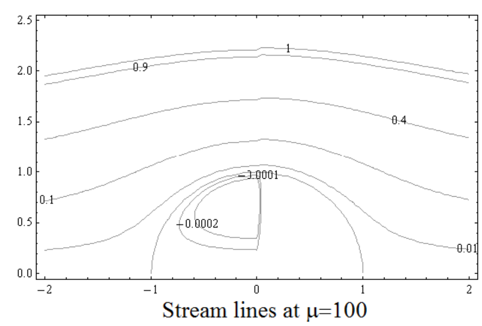

Fig. 6 (e). Stream lines for slip=10
$x_0 = 0$, Re=1 at μ =100

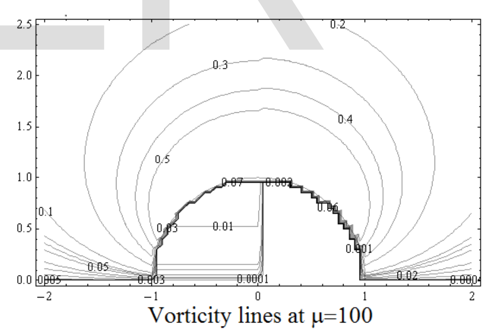

Fig. 6 (f). Vorticity lines for slip=10
$x_0 = 0$, Re=1 at μ =100

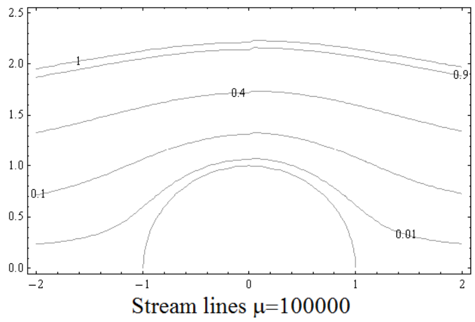

Fig. 6 (g). Stream lines for slip=10
$x_0 = 0$, Re=1 at μ =100000

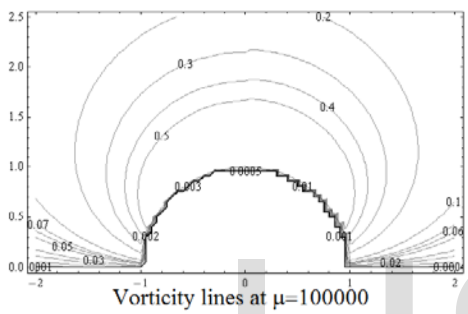

Fig. 6 (h). Vorticity lines for slip=10
$x_0 = 0$, Re=1 at μ =100000

6 CONCLUSION

In this manuscript, the stream function ψ and vorticity function Ω for uniform flow past a fluid sphere with contamination and slip boundary condition were found. Drag expression was obtained and as special cases (i) No slip condition as $s \to \infty$ and (ii) Solid sphere case as viscosity ratio $\mu \to \infty$ are evaluated. The deduced results are matching with the available standard results in the literature.
The flow pattern were drawn which are in good agreement with the results in literature.

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